Contents

How You May Use This Resource Guide

This guide is divided into chapters that match the chapters in the third editions of *Technical Mathematics* and *Technical Mathematics with Calculus* by John C. Peterson. The guide was originally developed for the second editions of these books by Robert Kimball, Lisa Morgan Hodge, and James A. Martin all of Wake Technical Community College, Raleigh, North Carolina. It has been modified for the third editions by the author.

Each chapter in this Resource Guide contains the objectives for that chapter, some teaching hints, guidelines based on NCTM and AMATYC standards, and activities. The teaching hints are often linked to activities in the Resource Guide, but also include comments concerning the appropriate use of technology and options regarding pedagogical strategies that may be implemented.

The guidelines provide comments from the *Crossroads* of the American Mathematical Association of Two-Year Colleges (AMATYC), and the Standards of the National Council of Teachers of Mathematics, as well as other important sources. These guidelines concern both content and pedagogy and are meant to help you consider how you will present the material to your students. The instructor must consider a multitude of factors in devising classroom strategies for a particular group of students. We all know that students learn better when they are actively involved in the learning process and know where what they are learning is used. We all say that less lecture is better than more lecture, but each one of us must decide on what works best for us as well as our students.

The activities provided in the resource guide are intended to supplement the excellent problems found in the text. Some activities can be quickly used in class and some may be assigned over an extended period to groups of students. Many of the activities built around spreadsheets can be done just as well with programmable graphing calculators; but we think that students should learn to use the spreadsheet as a mathematical tool. There are obstacles to be overcome if we are to embrace this useful technology for use in our courses, but it is worth the effort to provide meaningful experiences with spreadsheets to people who probably will have to use them on the job.

Whether or not you use any of the activities, we hope that this guide provides you with some thought-provoking discussion that will lead to better teaching and quality learning.

Chapter 11

Exponents and Radicals

Objectives

After completing this chapter, the student will be able to:

- Use basic rules for exponents to simplify exponential functions;
- Convert between radical form and exponential form;
- Use basic rules for radicals to simplify radical expressions;
- Add, subtract, multiply, and divide radical expressions;
- Solve equations involving radicals using both algebraic and graphical methods.

Teaching Hints

1. Review the rules for exponents from Chapter 1. Some students may need to be reminded of the rules. Make sure that students have a clear understanding of how to manipulate form to rewrite negative exponents into positive and vice versa.

2. Show students that radical functions are important and do occur in real life by modeling data using radical functions. Examples such as the length of a skid mark and the relationship between the oscillation time for a pendulum and the length of the pendulum arm are used in Activity 11.1.

3. Emphasize that when solving a radical equation algebraically it is better to isolate the most complicated radical first. Stress that when students square both sides they must square the entire side. Many students make the mistake of $(x+3)^2 = x^2+9$. This is a good place to review the shortcut for squaring a binomial, $(a+b)^2 = a^2 \pm 2ab + b^2$.

4. Emphasize the use of graphics in solving radical equations. Students often forget to check for extraneous solutions when they use algebraic methods for solving radical equations. Since graphical techniques will not show extraneous solutions if graphed from the original expression before squaring both sides, they can be used as a checking tool or as a solving tool. (see Activity 11.2)

5. The challenge of solving an equation creates the motivation for many algebraic topics. Equations make connections between mathematical topics. Examine equations containing radicals (as you did with all equations) using numerical investigation, graphical analysis and symbolic manipulation.

Guidelines

One of the content standards in *Crossroads* says:

> Students will use appropriate technology to enhance their mathematical thinking and understanding and to solve mathematical problems and judge the reasonableness of their results.

Students need to see how technology can be used to solve equations. The need to explore the benefits of using technology as well as the problems that can arise from using technology without understanding the concepts. In using a graphing utility to solve equations students will be able to get a visual understanding behind what is meant by solving an equation. The drill of manipulating radicals needs to be reduced, while the amount of time spent on applications involving radicals needs to be increased. Students need to be able to see the importance of the mathematical ideas that they study.

The *Crossroads* document also states that “Students will analyze data and use probability and statistical models to make inferences about real-world situations.” Students should look at data that can be modeled by the functions they are studying. If this is done as each type of function is encountered, then students will have a much easier time deciding from a scatter plot which type of function might best be modeled. The study of data sets that can be modeled with the functions being studied provides students with a reason for why each type of function must be studied.

Guidelines for Content

Increased Attention	Decreased Attention
Connection of functional behavior (such as where a function increases, decreases, achieves a maximum and/or minimum, or changes concavity) to the situation modeled by the function	Emphasis on the manipulation of complicated radical expressions, factoring, rational expressions, logarithms, and exponents
Graph theory and algorithms as a means of solving problems	Algebraically derived exact answers

Activities

1. Modeling Data with Radical Functions
   (a) Skidding
   (b) Pendulums
   In groups, students will explore data that can be modeled with radical functions.
2. Solving Radical Equations Graphically
   Students will use a graphing utility to solve radical equations.

Student Worksheet 11.1

Modeling Data with Radical Functions

The Highway Patrol measures skid marks at automobile accidents to help calculate the speed of the automobile. To make such a calculation, a formula must be found that relates speed and length of a skid mark under certain assumptions. Several factors other than speed could affect the length of the skid mark. For example, the weight of the auto could affect the length of the skid mark. Name several other factors that could affect the length of a skid mark in an automobile accident.

1. ____

2. ____

3. ____

Assume the data below was gathered by holding all factors constant except for speed. For example, only one size auto was used in the experiment to gather all the data below.

Skid (feet)	10	20	40	50	60	70	80
Speed (mph)	17.5	24.6	35.1	39.0	42.7	46.4	49.7

1. Plot the data on graph paper using $x =$ skid (ft) and $y =$ speed (mph). **2.** Find a formula that passes close to the data. **3.** Use your formula to predict the speed of an automobile if its skid mark is 120 feet long. **4.** Explain why this prediction could be valid or invalid.

2. Formula: ____

3. Prediction: ____

4. Explanation: ____

Student Worksheet 11.2

Pendulums: A Data Acquisition Activity

The relationship between the oscillation time for a pendulum and the length of the pendulum arm can be modeled by a radical function.

Materials:

Meter stick, string, weight, stopwatch

Procedure:

Attach the weight to the end of the string. Take oscillation time measurements (use four oscillations) for several different string lengths. Gather several data points

Data:

String length (cm)								
Oscillation time (sec)								

Plot the data on graph paper using x = string length (cm) and y = oscillation time (sec). Find a function that passes close to the data. Using your formula, predict the oscillation time for a string 125 cm long. Using the string, find the oscillation time for a string 125 cm long. How close was your predicted time to the actual time? What could cause the difference in the two times?

Possible Factors:

1. ______________________________
2. ______________________________
3. ______________________________

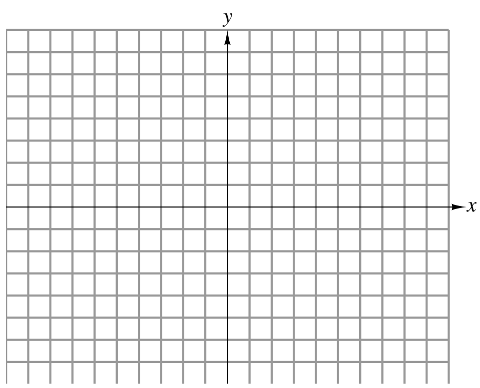
Student Worksheet 11.3

Solving Radical Equations Graphically

In Exercises 1–6, use a graphing utility to find the approximate solution of the radical equation. Sketch your graph on the grid provided and check your results in the equation. Be sure to graph the original expression, and do not square both sides to eliminate the radical.

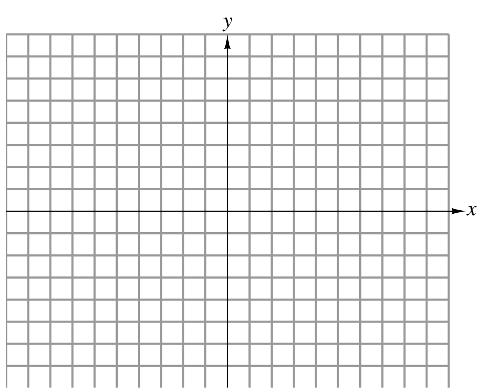
1. $\sqrt{x-1} = 2$

$x =$ ________



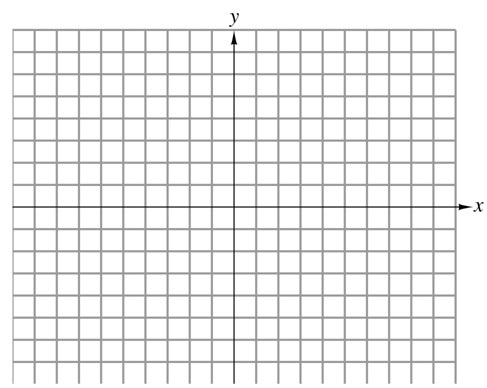
2. $\sqrt{x+7} = x - 5$

$x =$ ________



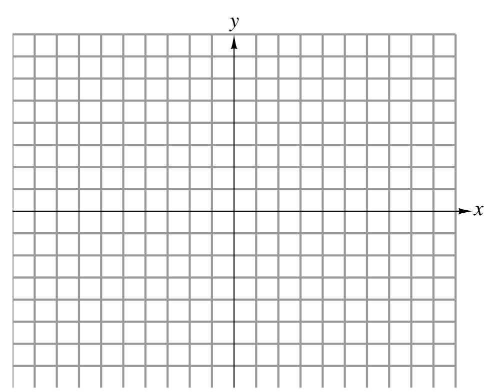
3. $2\sqrt{3x-2} = \sqrt{2x-3}$

$x =$ ________



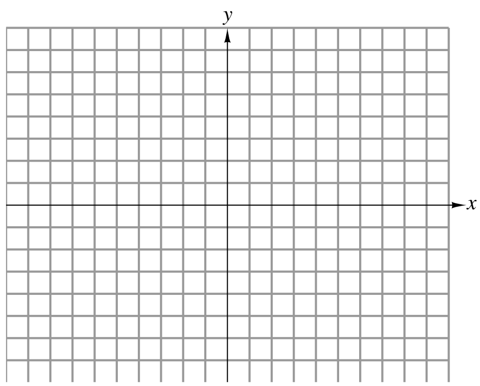
4. $2\sqrt{x-1} = x - 1$

$x =$ ________



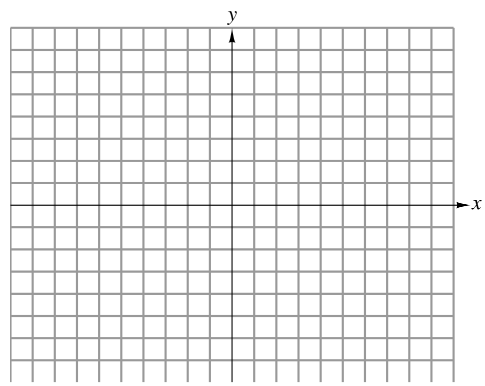
5. $\sqrt{x^2+5} - x + 2 = 0$

$x =$ ________



6. $\sqrt{x+8} - \sqrt{x} = 2$

$x =$ ________



7. Solve Exercise 1 algebraically and compare your results with your earlier graphical answer.

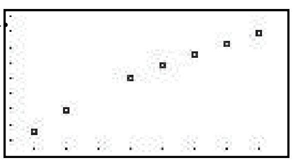
8. Solve Exercise 3 algebraically and compare your results with your earlier graphical answer.

9. Solve Exercise 4 algebraically and compare your results with your earlier graphical answer.

10. Write at least a three sentence paragraph comparing the two methods used: graphic and algebra. Discuss accuracy, the effort involved, the time spent and anything else you wish to compare.

Answers

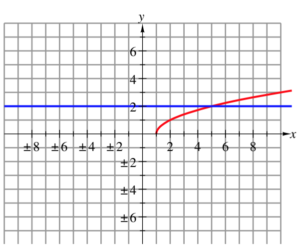
Student Worksheet 11.1

1. 

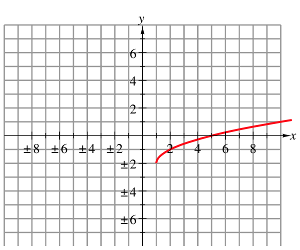
2. Skid $\approx 5.50\sqrt{x}$ feet when x is the speed in miles per hour

Student Worksheet 11.3

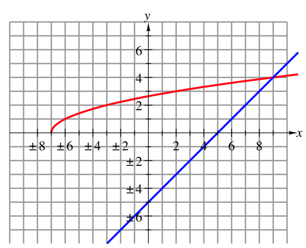
1. $x = 5$



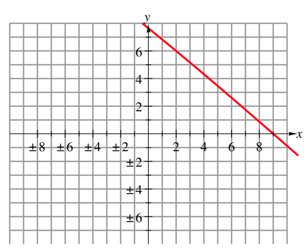
$y = \sqrt{x-1}$ and $y = 2$



$y = \sqrt{x-1} - 2$



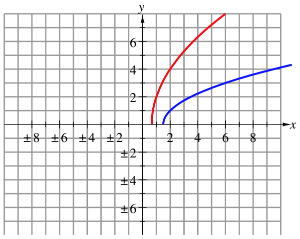
$y = \sqrt{x+7}$ and $y = x - 5$



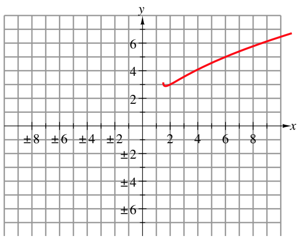
$y = \sqrt{x+7} - (x-5)$

2. $x = 9$

3. No real number solution

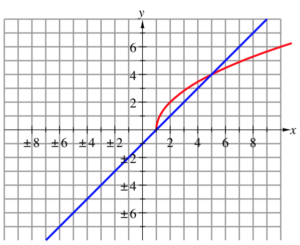


$y = 2\sqrt{3x-2}$ and $y = \sqrt{2x-3}$

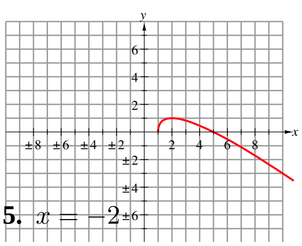


$y = 2\sqrt{3x-2} - \sqrt{2x-3}$

4. $x = 1$ or $x = 5$

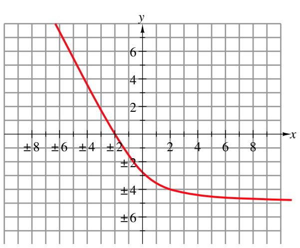


$2\sqrt{x-1}$ and $x - 1$



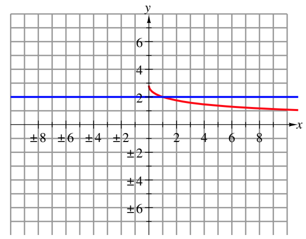
5. $x = -2$

$y = 2\sqrt{x-1} - (x-1)$

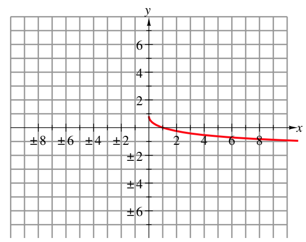


$y = \sqrt{x^2+5} - x - 5$

6. $x = 1$



$y = \sqrt{x+8} - \sqrt{x}$ and $y = 2$



$y = \sqrt{x+8} - \sqrt{x} - 2$

7.
$$\begin{aligned} \sqrt{x-1} &= 2 \\ x-1 &= 4 \quad Squarebothsides \\ x &= 5 \end{aligned}$$

The two answers are the same.

8.
$$\begin{aligned} 2\sqrt{3x-2} &= \sqrt{2x-3} \\ 4(3x-2) &= 2x-3 \quad Squarebothsides \\ 12x-8 &= 2x-3 \quad Expandterms \\ 10x &= 5 \quad Collectterms \\ x &= \tfrac{1}{2} = 0.5 \end{aligned}$$

But $x = 0.5$ is an extraneous root because it makes the $3x - 2 = -0.5$. Since this means that we need to take the square root of a negative number, there is no real number solution.

9.
$$\begin{aligned} 2\sqrt{x-1} &= x-1 \\ 4\,(x-1) &= x^2-2x+1 \quad Squarebothsides \\ 4x-4 &= x^2-2x+1 \quad Multiply \\ x^2-6x+5 &= 0 \quad Collectterms \\ (x-1)(x-5) &= 0 \quad Factor \\ x &= 1 \text{ or } 5 \end{aligned}$$

The two answers are the same.